Photodisintegration of Helium

M. L. RUSTGI AND S. N. MUKHERJEE
Physics Department, Banaras Hindu University, Varanasi, India
(Received 13 August 1962)

The electric dipole bremsstrahlung-weighted cross section σ_b and integrated cross section $\sigma_{\rm int}$ for the photodisintegration of helium have been calculated using the modified Irving wave function. Good agreement has been obtained with the current experimental values: $\sigma_b=2.4\pm0.15$ mb and $\sigma_{\rm int}=95\pm7$ MeV mb. The results have also been compared with the recent calculations of Goldhammer and Valk.

LET the electric dipole absorption cross section, neglecting retardation, of a nucleus for photons of energy W, averaged over all orientations of the nucleus, be denoted by $\sigma(W)$. The integrated bremsstrahlung-weighted cross section (σ_b) and the integrated cross section ($\sigma_{\rm int}$) are then defined by

$$\sigma_b=\int_0^\infty \frac{\sigma(W)}{W}dW, \tag{1}$$

$$\sigma_{\rm int}=\int_0^\infty \sigma(W)dW. \tag{2}$$

In their paper, Levinger and Bethe[1] had derived expressions for the electric dipole bremsstrahlung-weighted cross section and the integrated cross section for the nuclear photoeffect on the basis of the generalized Thomas-Reiche-Kuhn sum rule, using a partially attractive potential of Majorana type. The calculations of Levinger and Bethe were extended by Rustgi and Levinger[2] to include the two-body Heisenberg forces as well. By introducing the proton and neutron effective charges Levinger and Bethe obtained the following expressions for the bremsstrahlung-weighted cross section:

$$\sigma_b=\int_0^\infty \frac{\sigma}{W}dW=\frac{4\pi^2}{3}\frac{e^2}{\hbar c}\frac{NZ}{A}\langle r^2\rangle_{00}. \tag{3}$$

Here $\langle r^2\rangle_{00}$ denotes the mean square radius of the charge distribution of the nucleus. This expression, however, failed for small nuclei because terms of order $1/A$, neglected by Levinger and Bethe[1] in their use of effective charges, are appreciable for small nuclei. Foldy[3] has included these terms and obtained

$$\sigma_b=\tfrac{4}{3}\pi^2(e^2/\hbar c)[NZ/(A-1)]\langle r^2\rangle_{00}, \tag{4}$$

where

$$\langle r^2\rangle_{00}=(1/Z)[\textstyle\sum_i(r_i-R)^2]_{00}. \tag{5}$$

Here "i" stands for the proton and R is the coordinate of the center of mass of the nucleus. Foldy's derivation assumes a nuclear wave function that is spatially symmetric for all pairs of nucleons and should be quite accurate for $A=2$, 3, and 4. The special cases of $A=3$ and $A=4$ were treated by Rustgi[4] and Rustgi and Levinger,[2] using Irving's wave function.[5] A comparison between theory and experiment for the He^4 nucleus showed a serious discrepancy between the two for the bremsstrahlung-weighted cross section (σ_b) and the agreement between the two for the integrated cross section ($\sigma_{\rm int}$) was also not so good. Since the calculations of σ_b usually provide a good check on the ground-state wave function, the large disagreement between theory and experiment for σ_b was thought to be due to the small rms value of the alpha particle given by Irving's[5] wave function. Recently, however, new experimental data have become available through the work of Gorbunov and Spiridonov[6] and the wave function of Irving has also been modified by one of the authors[7] to fit the size and binding energy of the He^4 nucleus. It was, therefore, thought desirable to re-evaluate both the electric dipole bremsstrahlung-weighted cross section and the integrated cross section to find out if the modified Irving wave function could resolve the discrepancy between experiment and theory. The present treatment for the integrated cross section differs from that of Rustgi and Levinger and has been, therefore, described in detail.

The complete wave function for the ground state of an alpha particle representing a mixture of the ${}^1S_0$ and the principal ${}^5D_0$ states may be written in the form[2,5]

$$\psi=[1/(1+C^2)^{1/2}](\psi_S+C\psi_D), \tag{6}$$

where

$$\psi_S=N_S\exp[-2\alpha(u^2+v^2+w^2)^{1/2}], \tag{7}$$

$$\psi_D=N_D\exp[-2\beta(u^2+v^2+w^2)^{1/2}][6(\boldsymbol{\sigma}_1\cdot\mathbf{v})(\boldsymbol{\sigma}_3\cdot\mathbf{w})+6(\boldsymbol{\sigma}_1\cdot\mathbf{w})(\boldsymbol{\sigma}_3\cdot\mathbf{v})-4(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)(\mathbf{v}\cdot\mathbf{w})], \tag{8}$$

$$\mathbf{v}=(\mathbf{r}_2-\mathbf{r}_1)/2^{1/2};\quad \mathbf{w}=(\mathbf{r}_4-\mathbf{r}_3)/2^{1/2}, \tag{9}$$

$$\mathbf{u}=\tfrac{1}{2}(\mathbf{r}_4+\mathbf{r}_3-\mathbf{r}_2-\mathbf{r}_1), \tag{10}$$

$$\mathbf{R}=\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4), \tag{11}$$

$$N_S^2=2^6\alpha^9/3\pi^4;\quad N_D^2=2^7\beta^{13}/(5^2 3^4\pi^4), \tag{12}$$

and

$$r_{12}^2+r_{13}^2+r_{14}^2+r_{23}^2+r_{34}^2+r_{24}^2=4(u^2+v^2+w^2). \tag{13}$$

[1] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[2] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).
[3] L. L. Foldy, Phys. Rev. **107**, 1303 (1957).
[4] M. L. Rustgi, Phys. Rev. **106**, 1256 (1957).
[5] J. Irving, Proc. Phys. Soc. (London) **A66**, 17 (1953).
[6] A. Gorbunov and V. Spiridonov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 21 (1957); **34**, 862, 866 (1958).
[7] S. N. Mukherjee, Nuovo Cimento **25**, 509 (1962).

Here $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$ denote the positions of the particles, 1 and 2 denote the neutron and 3,4 the proton coordinates. C^2 denotes the amount of "D" state in the mixtures assuming that "S" and "D" parts of the wave function have been separately normalized to unity. α, β, and C are the parameters that are usually determined by a variational calculation. Instead, Mukherjee[7] has used the high-energy $e-\mathrm{He}^4$ scattering experiments[8] to determine α and, employing a potential of the form

$$V(r_{ij})=-V_0\left([1-\tfrac{1}{2}g+\tfrac{1}{2}g(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)]\frac{\exp(-r_{ij}/r_c)}{r_{ij}/r_c}+\gamma S_{ij}\frac{\exp(-r_{ij}/r_t)}{r_{ij}/r_t}\right),$$

has determined the other two parameters β and C by a variational calculation. He finds that

$$\alpha=0.841\times10^{13}\ \mathrm{cm}^{-1},\quad \beta=1.365\times10^{13}\ \mathrm{cm}^{-1},$$

and

$$C=-0.153. \tag{14}$$

Following Rustgi and Levinger, one can write for the bremsstrahlung-weighted cross section using the modified Irving wave function,

$$\sigma_b=\int(\sigma/W)dW \tag{15}$$

$$=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{1}{(1+C^2)}\left[\left(\frac{15}{8\alpha^2}\right)+\left(\frac{21C^2}{8\beta^2}\right)\right]=2.70\ \mathrm{mb}. \tag{16}$$

This value is considerably higher than the one obtained by Rustgi and Levinger and is in reasonably good agreement with the experimental value (2.4 ± 0.15) mb obtained by Gorbunov and Spiridonov.[6]

In the sum-rule calculations of Levinger and Bethe and Rustgi and Levinger it is shown that for central forces of the form $V(r_{ij})[1+xP_{ij}^M+yP_{ij}^H]$, the integrated cross section may be written as

$$\sigma_{\mathrm{int}}=\frac{2\pi^2e^2\hbar}{MC}\left[\frac{NZ}{A}-\frac{Mx}{3\hbar^2}\int\psi_0^*\sum_i\sum_j V(r_{ij})r_{ij}^2P_{ij}^M\psi_0d\tau-\frac{My}{3\hbar^2}\int\psi_0^*\sum_i\sum_j V(r_{ij})r_{ij}^2P_{ij}^H\psi_0d\tau\right], \tag{17}$$

where the summations extend over i protons and j neutrons, and x and y are the fractions of Majorana and Heisenberg exchange forces present in the two-body interaction operator $V(r_{ij})$. The other notation follows that of reference 2. For the present calculation a potential of the following form will be assumed:

$$V(r_{ij})=-V_0[(a+bP_{ij}^B+xP_{ij}^M+yP_{ij}^H)J(r_{ij})+\gamma(a'+b'P_{ij}^B+x'P_{ij}^M+y'P_{ij}^H)K(r_{ij})S_{ij}], \tag{18}$$

where the tensor operator is

$$S_{ij}=r_{ij}^{-2}[3(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{ij})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ij})-(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)], \tag{19}$$

and

$$J(r_{ij})=\frac{\exp(-r_{ij}/r_c)}{(r_{ij}/r_c)},\quad K(r_{ij})=\frac{\exp(-r_{ij}/r_t)}{(r_{ij}/r_t)}. \tag{20}$$

Here the parameters V_0, γ, r_c, and r_t have been chosen to fit the binding energy of the helium nucleus.[7]

$$V_0=54.59\ \mathrm{MeV},\quad \gamma=0.2305,\quad r_c=1.184\times10^{-13}\ \mathrm{cm},\quad r_t=2.121\times10^{-13}\ \mathrm{cm}. \tag{21}$$

For an alpha particle,

$$\sigma_{\mathrm{int}}=\frac{2\pi^2e^2\hbar}{Mc}\left[1+\frac{MV_0}{(1+C^2)3\hbar^2}\int(\psi_S^*+C\psi_D^*)\times\{\sum_i\sum_j r_{ij}^2[(xP_{ij}^M+yP_{ij}^H)J(r_{ij})+(x'P_{ij}^M+y'P_{ij}^H)\gamma K(r_{ij})S_{ij}]\}(\psi_S+C\psi_D)d\tau\right]. \tag{22}$$

For the "S" state of an even-even nucleus

$$\psi_S^*P^H\psi_S=\psi_S^*P^MP^B\psi_S=\tfrac{1}{2}\psi_S^*P^M\psi_S. \tag{23}$$

Further,

$$P_{ij}^BS_{ij}=S_{ij}P_{ij}^B=S_{ij}, \tag{24}$$

and

$$\psi_D^*P_{ij}^MP_{ij}^B\psi_D=\psi_D^*P_{ij}^M\psi_D. \tag{25}$$

From the orthogonality of the "S" and "D" states, it follows that

$$\sigma_{\mathrm{int}}=\frac{2\pi^2e^2\hbar}{Mc}\left\{1+\frac{MV_0}{(1+C^2)3\hbar^2}\left[(x+\tfrac{1}{2}y)\int\psi_S^*\sum_i\sum_j J(r_{ij})r_{ij}^2\psi_Sd\tau+2C\gamma(x'+y')\int\psi_S^*\sum_i\sum_j K(r_{ij})S_{ij}r_{ij}^2\psi_Dd\tau+C^2(x+y)\int\psi_D^*\sum_i\sum_j J(r_{ij})r_{ij}^2\psi_Dd\tau+C^2\gamma(x'+y')\int\psi_D^*\sum_i\sum_j K(r_{ij})S_{ij}r_{ij}^2\psi_Dd\tau\right]\right\}. \tag{26}$$

[8] R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956).

This expression differs from that of Rustgi and Levinger[2] [Eq. (24) of reference 2] in the third, fourth, and fifth terms. The differences arise because Rustgi and Levinger assumed Eq. (23) to be valid even for the "D" state. The spin matrix elements can now be evaluated as in Irving's paper. The spatial integrals have the general form

$$\int_0^1 \frac{t^p(1-t^2)^q dt}{(a+t)^r} = \frac{1}{(a+1)^{p+1}} \frac{1}{a^{q+1}} \sum_{\mu=0}^{q} \sum_{\nu=\mu}^{\mu+n-q} \frac{B(p+\nu+1, r+n-\nu)}{a^{n-\nu}(a+1)^{\nu}} \, {}^qC_{\mu} \, {}^{n-q}C_{\nu-\mu},$$

where

$$n=r-p-q-2,\ a>0,\ p+1>0,\ q+1>0,\ r\geqslant p+2q+2,$$

$${}^nC_r = \frac{n!}{r!(n-r)!}, \quad B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}. \tag{27}$$

On working out the radial integrals and on substituting the value of the constants, one obtains for the integrated cross section

$$\sigma_{\text{int}} = 60[1+0.635(x+\tfrac{1}{2}y)+0.124(x'+y') +0.014(x+y)]. \tag{28}$$

Table I shows the values of σ_{int} for the Rosenfeld, Inglis, and Serber mixtures, assuming the same parameters for the central and tensor forces ($x=x'$, $y=y'$).

The result for the bremsstrahlung-weighted cross section $\sigma_b = 2.70$ mb is in complete agreement with the calculations of Goldhammer and Valk but there is a 25% disagreement in the value of the integrated cross section

TABLE I. Integrated cross sections for photodisintegration of the helium nucleus.

Mixture	a	x	b	y	σ_{int} (MeV mb)
Rosenfeld	−0.13	0.93	0.46	−0.26	96.0
Inglis	0	0.8	0.2	0	97.1
Serber	0.5	0.5	0	0	83.2

as calculated for the Serber mixture. This disagreement may be mainly attributed to the unusually large "D"-state mixture of about 10.6% employed by Goldhammer and Valk to fit the binding energy of the helium nucleus. The presence of this large "D"-state probability enhances the tensor contribution to the integrated cross section as shown in Table II.

TABLE II. Contributions to the integrated cross section for helium.

Type of contribution	Goldhammer and Valk[a] (MeV-mb)	Present work (MeV-mb)
Nonexchange	60.0	60.0
Total central	27.7	19.5
Total tensor	19.7	3.7
Total	107.4	83.2

[a] P. Goldhammer and H. S. Valk [Phys. Rev. **127**, 945 (1962)], have evaluated the integrated cross section only for the Serber mixture. Calculations with other force mixtures would have further enhanced the disagreement for σ_{int} between their calculations and the experimental value $\sigma_{\text{int}} = 95 \pm 7$ MeV mb.

The results of the present calculations for σ_b and σ_{int} for Rosenfeld and Inglis mixtures are in good agreement with the experimental values. Perhaps another interesting number for comparison with experiments is the harmonic mean energy $W_H = \sigma_{\text{int}}/\sigma_b$. Using $\sigma_{\text{int}} = 95 \pm 7$ MeV mb and $\sigma_b = 2.4 \pm 0.15$ mb, the experiments give $W_H = 39.6$ MeV. The results of the present calculations give $W_H = 36$ MeV, which is also in reasonable agreement with experiments.

ACKNOWLEDGMENT

The authors are grateful to Professor A. R. Verma for providing the facilities to carry out this work.